

Formulas for Math 9B

AP_{EX} CALCULUS

Integral Calculus

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Index of Tables of Review Formulas:

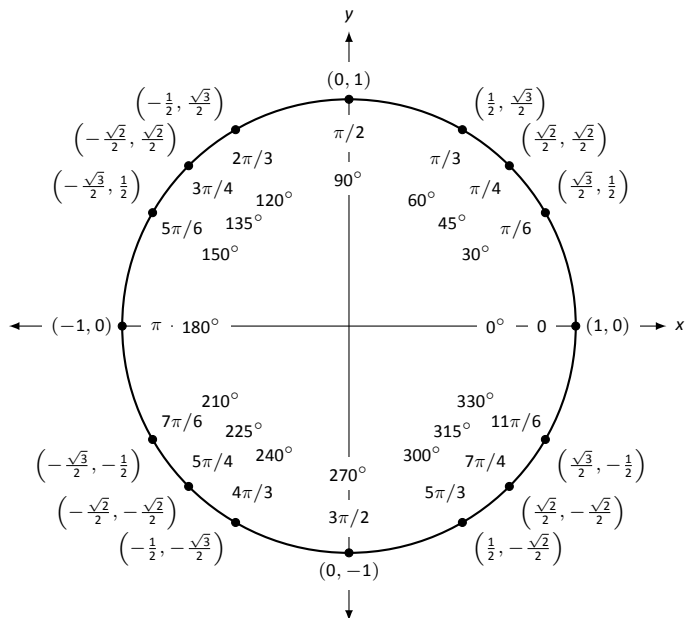
Derivative Rules
Trigonometry
Areas & Volumes
Basic Algebra

Differentiation Rules (Math 9A)

- | | | | |
|---|--|--|---|
| 1. $\frac{d}{dx}(cx) = c$ | 10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$ | 19. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ | 28. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ |
| 2. $\frac{d}{dx}(u \pm v) = u' \pm v'$ | 11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$ | 20. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ | 29. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ |
| 3. $\frac{d}{dx}(u \cdot v) = uv' + u'v$ | 12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$ | 21. $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{ x \sqrt{x^2-1}}$ | 30. $\frac{d}{dx}(\operatorname{coth} x) = -\operatorname{csch}^2 x$ |
| 4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$ | 13. $\frac{d}{dx}(\sin x) = \cos x$ | 22. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{ x \sqrt{x^2-1}}$ | 31. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ |
| 5. $\frac{d}{dx}(u(v)) = u'(v)v'$ | 14. $\frac{d}{dx}(\cos x) = -\sin x$ | 23. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ | 32. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$ |
| 6. $\frac{d}{dx}(c) = 0$ | 15. $\frac{d}{dx}(\csc x) = -\csc x \cot x$ | 24. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$ | 33. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$ |
| 7. $\frac{d}{dx}(x) = 1$ | 16. $\frac{d}{dx}(\sec x) = \sec x \tan x$ | 25. $\frac{d}{dx}(\cosh x) = \sinh x$ | 34. $\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{ x \sqrt{1+x^2}}$ |
| 8. $\frac{d}{dx}(x^n) = nx^{n-1}$ | 17. $\frac{d}{dx}(\tan x) = \sec^2 x$ | 26. $\frac{d}{dx}(\sinh x) = \cosh x$ | 35. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ |
| 9. $\frac{d}{dx}(e^x) = e^x$ | 18. $\frac{d}{dx}(\cot x) = -\csc^2 x$ | 27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ | 36. $\frac{d}{dx}(\operatorname{coth}^{-1} x) = \frac{1}{1-x^2}$ |

Trigonometry

The Unit Circle



Common Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Angle Sum/Difference Formulas

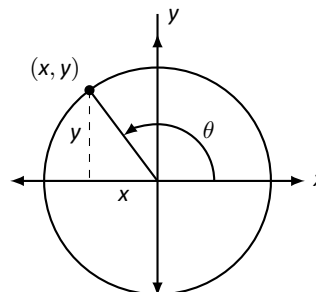
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Definitions of the Trigonometric Functions

Unit Circle Definition

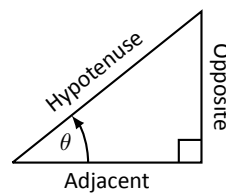


$$\sin \theta = y \quad \cos \theta = x$$

$$\csc \theta = \frac{1}{y} \quad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

Right Triangle Definition



$$\sin \theta = \frac{O}{H} \quad \csc \theta = \frac{H}{O}$$

$$\cos \theta = \frac{A}{H} \quad \sec \theta = \frac{H}{A}$$

$$\tan \theta = \frac{O}{A} \quad \cot \theta = \frac{A}{O}$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Other Trigonometric Identities

Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

Areas and Volumes

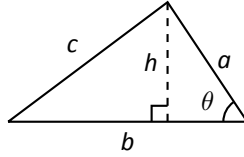
Triangles

$$h = a \sin \theta$$

$$\text{Area} = \frac{1}{2}bh$$

Law of Cosines:

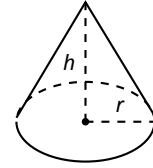
$$c^2 = a^2 + b^2 - 2ab \cos \theta$$



Right Circular Cone

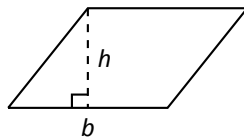
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \pi r \sqrt{r^2 + h^2} + \pi r^2$$



Parallelograms

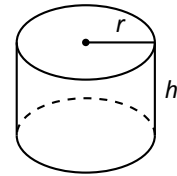
$$\text{Area} = bh$$



Right Circular Cylinder

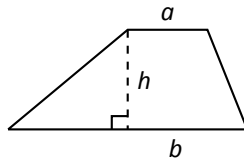
$$\text{Volume} = \pi r^2 h$$

$$\text{Surface Area} = 2\pi rh + 2\pi r^2$$



Trapezoids

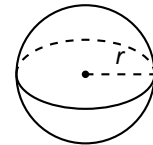
$$\text{Area} = \frac{1}{2}(a + b)h$$



Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

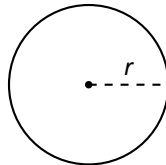
$$\text{Surface Area} = 4\pi r^2$$



Circles

$$\text{Area} = \pi r^2$$

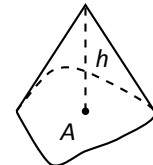
$$\text{Circumference} = 2\pi r$$



General Cone

$$\text{Area of Base} = A$$

$$\text{Volume} = \frac{1}{3}Ah$$

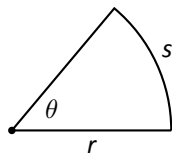


Sectors of Circles

θ in radians

$$\text{Area} = \frac{1}{2}\theta r^2$$

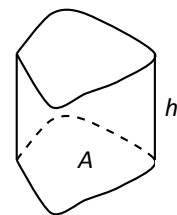
$$s = r\theta$$



General Right Cylinder

$$\text{Area of Base} = A$$

$$\text{Volume} = Ah$$



Basic Algebra

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \leq b^2 - 4ac$, then the real zeros of p are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Special Factors

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \dots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \dots \pm nxy^{n-1} \mp y^n$$

Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$$

Factoring by Grouping

$$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$$

Arithmetic Operations

$$ab + ac = a(b + c)$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\left(\frac{a}{b}\right) \left(\frac{c}{d}\right) = \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) = \frac{ac}{bd}$$

$$\left(\frac{a}{b}\right) \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$a \left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{ab + ac}{a} = b + c$$

Exponents and Radicals

$$a^0 = 1, \quad a \neq 0$$

$$(ab)^x = a^x b^x$$

$$a^x a^y = a^{x+y}$$

$$\sqrt{a} = a^{1/2}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$a^{-x} = \frac{1}{a^x}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$(a^x)^y = a^{xy}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$